Violation of a Bell inequality in two-dimensional orbital angular momentum state-spaces

J. Leach[1], B. Jack[1], J. Romero[1], M. Ritsch-Marte[2], R. W. Boyd[3], A. K. Jha[3], S. M. Barnett[4], S. Franke-Arnold[1] and M. J. Padgett[1]

[1] *Department of Physics and Astronomy, SUPA, University of Glasgow, Glasgow, UK*

[2] *Division for Biomedical Physics, Innsbruck Medical University, Innsbruck, Austria*

[3] *Institute of Optics, University of Rochester, Rochester, NY 14627 USA*

[4] *Department of Physics, SUPA, University of Strathclyde, Glasgow, UK*
j.leach@physics.gla.ac.uk

Abstract: We observe entanglement between photons in controlled superposition states of orbital angular momentum (OAM). By drawing a direct analogy between OAM and polarization states of light, we demonstrate the entangled nature of high order OAM states generated by spontaneous downconversion through violation of a suitable Clauser Horne Shimony Holt (CHSH)-Bell inequality. We demonstrate this violation in a number of two-dimensional subspaces of the higher dimensional OAM Hilbert space.

OCIS codes: (270.0270) Quantum optics; (070.6120) Spatial light modulators

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Entanglement is the quintessential quantum phenomenon, associated with the non-classical correlations between separate quantum systems. It is viewed increasingly as a resource for quantum communications and the processing of quantum information [1]. Quantifying the degree of entanglement is a pressing challenge for quantum information theory [2]. Polarization formed the basis of much of the early work on quantum entanglement and related paradoxes [3, 4]. Since then, other experiments have been developed, including correlation measurements between time and energy [5], momentum and position [6, 7], spatial distributions [8, 9, 10] and the orbital angular momentum of light [11, 12, 13, 14, 15, 17, 18, 19].

It is now well recognized that in addition to the spin angular momentum, characterized by its polarization, light may carry orbital angular momentum (OAM). The OAM eigenstates are characterized by helical phasefronts described by $\exp(i\ell\phi)$, where ϕ is the azimuthal angle

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[20]. It is possible to construct matrices that describe transformations and superpositions of orthogonal OAM states [21] and to project these onto two-dimensional subspaces that can be represented by a Bloch sphere, equivalent to the Poincaré sphere for polarization [22, 19], see Fig. 1. Unlike polarization, which is described within a two-dimensional Hilbert-space, OAM has an unbounded number of orthogonal states potentially offering a way of inscribing quantum information in a high-dimensional basis [23].

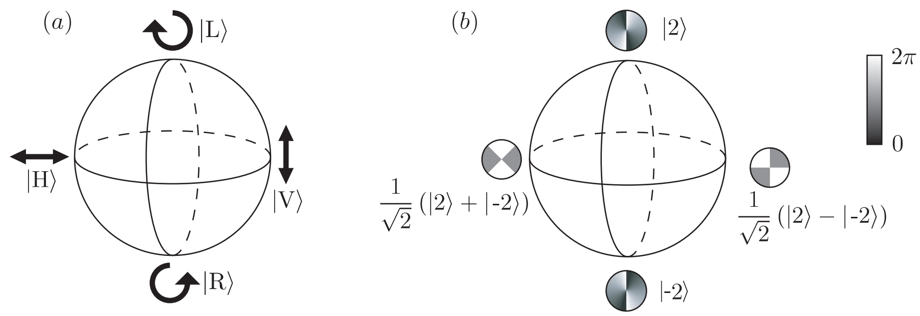

Fig. 1. Bloch spheres for (a) polarization and (b) OAM ($|\pm 2\rangle$) states. The sphere for OAM states shows the phase structure (grayscale) of the relevant modes. The states on the sphere are expressed solely in terms of their azimuthal phase (i.e. ℓ-value) without any reference to their radial dependence.

In 2001 Zeilinger and co-workers reported the first correlation measurements on OAM states. They showed that conservation of the transverse momentum within parametric downconversion leads to a correlation between the OAM states of the signal and idler beams, which they measured holographically [12]. Deliberate displacement of the holograms showed that these correlations persisted for superposition states, indicative of quantum entanglement. The same group demonstrated a violation of a Bell inequality in a three-dimensional OAM subspace again by using displaced holograms [13]. Subsequent work by Woerdman and co-workers has shown correlations between fractional OAM states as measured using angular phaseplates [15, 16]. These phaseplates have also been used to determine the number of dimensions over which the OAM is entangled [17]. OAM entanglement has been combined with polarization and energy-time entanglement to produce hyper-entangled photon pairs, with the entanglement of each degree of freedom, demonstrated by violation of a suitable Bell inequality [18]. Most recently we have demonstrated a Fourier-relationship between the angle and angular momentum of the entangled photons. Defining the angular distribution of one photon influences, if measured in coincidence, the OAM spectrum of the other. The coincidence count rate is determined by the Fourier transform of the aperture function [24].

In this letter, we utilize and extend the analogy that exists between polarization and OAM to two-photon entanglement. By using computer designed holograms, we can generate arbitrary two-photon states entangled within a two-dimensional OAM subspace. This reduction to two-dimensions, and hence to dichotomous variables, is essential in order to complete the analogy with polarization and to demonstrate a conflict with the assumptions underlying Bell's inequality. We demonstrate the entangled nature of these OAM superposition states through violation of a suitable Clauser Horne Shimony Holt (CHSH)-Bell inequality.

In earlier work [22], the modified Poincaré sphere was used to describe Laguerre-Gaussian modes with $\ell = \pm 1$ which, when added in equal weights, give a Hermite-Gaussian mode. In that case, the surface of the sphere maps out all possible superpositons of these modes, i.e. a 2-dimensional OAM subspace. This approach can be extended to encompass superpositions of helically phased modes of any order. For a given order, modes $|+\ell\rangle$ and $|-\ell\rangle$ are represented

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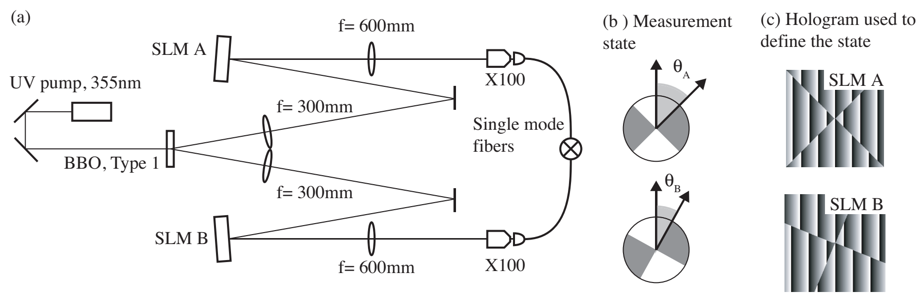

Fig. 2. Schematic of experimental apparatus for using spatial light modulators to measure the correlations in orbital states of down-converted photons

by the north and the south poles on the Bloch sphere. An equally weighted superposition of $|+\ell\rangle$ and $|-\ell\rangle$ with an arbitrary relative phase is represented by a point along the equator. In an analogous fashion to the polarization states, the superpositions given by

$$|\theta\rangle = \frac{1}{\sqrt{2}}(e^{i\ell\theta}|+\ell\rangle + e^{-i\ell\theta}|-\ell\rangle) \tag{1}$$

are represented by points along the equator. For the special case where $\ell = 1$, these modes are rotated Hermite-Gaussian modes, where the angle θ relates to their orientation. More generally, the $|\theta\rangle$ modes are not pure Hermite-Gaussian modes but they have 2ℓ sectors of alternating phases [25], and in this paper we refer to these as *s*ector states. Figure 1b shows the phase distribution of a sector state formed by $|+2\rangle$ and $|-2\rangle$ modes. These sector states are equivalent to the linear polarization states that lie on the equator of the Poincaré sphere. The analogy between polarization and OAM suggests a methodology for investigating entanglement in OAM as others have done previously for polarization [3, 4].

In spontaneous parametric downconversion (SPDC) a pump photon gets down-converted to a pair of signal and idler photons. When the pump is a Gaussian beam, the state of the two-photon field produced by SPDC can be written as [26]:

$$|\Psi\rangle = \sum_{n=-\infty}^{n=+\infty} c_n |n\rangle_s |-n\rangle_i. \tag{2}$$

Here s and i stand for signal and idler respectively; $|n\rangle$ denotes the OAM eigenmode of order n and $|c_n|^2$ is the probability to generate a photon pair with OAM $\pm n$. Due to conservation of angular momentum, if the signal photon is in mode given by $|n\rangle$ then idler photon can only be in mode given by $|-n\rangle$, with an associated probability amplitude c_n [12, 28, 27]. From the symmetry of the SPDC process, we can infer that $c_n = c_{-n}$.

In order to quantify the entanglement, we must demonstrate that correlations of the signal and idler persist for superposition states. For this, we detect the photons in the sector states defined by Eq. 1 oriented at different angles θ_A and θ_B respectively. Using Eq.(1) and (2), the coincidence rate $C(\theta_A, \theta_B)$ for detecting one photon in sector state $|\theta_A\rangle$ and the other in $|\theta_B\rangle$ is then given by

$$C(\theta_A, \theta_B) = |\langle\theta_A|\langle\theta_B||\Psi\rangle|^2 \propto \cos^2[\ell(\theta_A - \theta_B)]. \tag{3}$$

The high-visibility sinusoidal fringes of this joint probability are the signature of two-dimensional entanglement. Any modal impurities will degrade the entangled nature of the state such that the visibility of the fringes will would be reduced.

Figure 2 shows our experimental arrangement. Our down-conversion source is a frequency-tripled, mode-locked Nd-YAG laser with an average output power of 150 mW at 355 nm.

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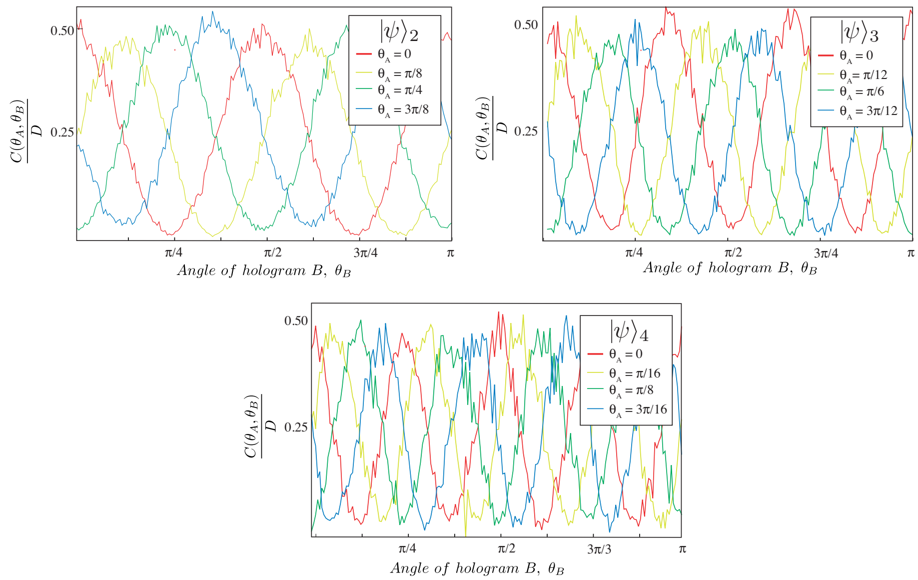

Fig. 3. The relative coincidence count as a function of relative orientation of the sector apertures. The measured count rates were normalized by D, the denominator in equ 6. It is the coincidence counts at orientations $\theta_A = 0, \theta_B = \frac{\pi}{8\ell}, \theta'_A = \frac{\pi}{4\ell}$ and $\theta'_B = \frac{3\pi}{8\ell}$ which show the largest violation of the Bell inequality.

The collimated output beam is normally incident on a 3 mm long BBO crystal, which is cut for degenerate type-I non-collinear phase-matching with a semi-cone angle for the down-converted beams of 4°. The single-photon detection is accomplished using spatial light modulators (SLMs, Hamamatsu), which are used to define the sector state by modifying the spatial profile of the signal and idler beams, followed by avalanche photo-diodes coupled to single-mode fibers. The fiber facets are imaged to the plane of the SLMs which in turn are imaged onto the non-linear crystal so that they overlap with each other and are centered on the pump beam. If one considers the single-mode fibers as "mode-projectors", the ratio between the waist of the pump beam upon the crystal and the waist of the single-mode fiber projected to this plane approximately 1. This ratio is chosen in order to maximise the collection of the down-converted light. The TTL pulses from the two detectors are fed to a counter with a time resolution $\Delta t = 25$ *ns*, from which the coincidence counts are logged.

The sector states are experimentally defined by phase apertures on the SLMs. These phase apertures serve to select the two-dimensional subspace and act as analyzers for these states when their relative orientation is changed. As the SLMs are used as phase only devices and do not control the intensity profile of the beam, the sector states they define generate small contributions associated with higher order ℓ components. However as the angular distribution of modes generated in the down conversion process is limited, the contribution from these higher order sidebands is extremely low. The two-photon state post-selected by the two angular apertures is very close that desired and can therefore be approximated by:

$$|\psi\rangle_\ell = \frac{1}{\sqrt{2}}[|\ell\rangle_s|-\ell\rangle_i + |-\ell\rangle_s|\ell\rangle_i] \tag{4}$$

The above state is normalized within the chosen subspace and is entangled in OAM. By choosing the number of sectors in a phase aperture an entangled state with any given ℓ can be pre-

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pared.

Figure 3 shows the recorded coincidence fringes for the entangled states, $|\psi\rangle_2, |\psi\rangle_3$ and $|\psi\rangle_4$ where the phase aperture on SLM B is rotated with respect to the phase aperture on SLM A. The angle between our phase patterns is equivalent to the angle between polarizers used in experiments that demonstrate a Bell inequality violation for polarization. By fixing the angle θ_A and scanning the hologram in channel B for angles $\theta_B = 0$ to 2π we observe a sinusoidally varying coincidence rate, which is the signature of Bell-type experiments, see Fig 3. The ℓ-dependance of the 2-dimensional entangled state shows up in the periodicity of the recorded fringe patterns. While for polarisation, the repetition angle is π, in our case, the periodicity is π/ℓ.

The sinusoidal behaviour of the coincidence rate follows directly from quantum mechanics by overlapping different sector states, see Eq. 3. Bell's work proved that such behaviour can not be simulated by any classical theory based on local hidden-variables [29]. Interestingly, correlations between orthogonal states can be simulated by classical correlations, whereas correlations between certain superposition states can be stronger than classically allowed. The deviation of a quantum experiment from classical local theory can be measured in terms of Bell's inequality. A commonly used variant of Bell's inequality is the symmetrised version devised by Clauser, Horne and Shimony and Holt (CHSH) [30, 31, 32]. For our experiments, where θ_A and θ_B are the angles of the phase masks on the SLMs, we define the Bell parameter S to be,

$$S = E(\theta_A, \theta_B) - E(\theta_A, \theta_B') + E(\theta_A', \theta_B) + E(\theta_A', \theta_B') \tag{5}$$

The inequality is violated for values of $|S|$ which are greater than 2. $E(\theta_A, \theta_B)$ is calculated from the coincidence rates at particular orientations,

$$E(\theta_A, \theta_B) = \frac{C(\theta_A, \theta_B) + C(\theta_A + \frac{\pi}{2\ell}, \theta_B + \frac{\pi}{2\ell}) - C(\theta_A + \frac{\pi}{2\ell}, \theta_B) - C(\theta_A, \theta_B + \frac{\pi}{2\ell})}{C(\theta_A, \theta_B) + C(\theta_A + \frac{\pi}{2\ell}, \theta_B + \frac{\pi}{2\ell}) + C(\theta_A + \frac{\pi}{2\ell}, \theta_B) + C(\theta_A, \theta_B + \frac{\pi}{2\ell})}. \tag{6}$$

For a given entangled state $|\psi\rangle_\ell$, the inequality is maximally violated for example when $\theta_A = 0, \theta_B = \frac{\pi}{8\ell}, \theta_A' = \frac{\pi}{4\ell}$ and $\theta_B' = \frac{3\pi}{8\ell}$. Table I gives the measured values of S at these angles and shows a clear violation of the inequality for all the measured subspaces. We should point out at that our experimental configuration does not satisfy the conditions required to avoid the remote measurement nor fair sampling loop-holes. Whilst the motivation for the experiment was not concerned with these loop-holes, improvements the overall quantum efficiencies (detectors and SLMs) and the location of our detectors could allow us to address these issues.

Table 1. Different entangled states $|\psi\rangle_l$ and the associated measured value of S. A value of $|S| > 2$ violates the CHSH inequality.

Entangled state	S	Violation by σ
$\|\psi\rangle_2$	2.69 ± 0.02	35
$\|\psi\rangle_3$	2.55 ± 0.04	14
$\|\psi\rangle_4$	2.33 ± 0.07	5

We have observed entanglement between sector states which are superpositions of OAM states. By drawing a direct analogy between polarization and OAM we have demonstrated a violation of a Bell inequality for various 2-dimensional subspaces of the unbounded OAM Hilbert space by up to 35 standard deviations. Bell inequalities have been formulated for higher-dimensional spaces [33, 13] and entangled OAM states provide an ideal system with which to test these more general inequalities. Our technique allows for the precise control of the preparation of entangled two-photon states relevant to quantum computing and telecommunications systems [14, 34].

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